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# ELIMINATION OF EDGE RUPTURE CAUSED BY THE EFFECT OF FLOW PULSATIONS*

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The possibility of eliminating boundary-layer (BL) separation at the leading edge of a slender profile is studied using high-frequency harmonic pulsations in the velocity of the oncoming flow near some mean value. The pulsation period is assumed to be small since the Strouhal number is much greater than 1. In this case the BL is divided into a narrow non-stationary Stokes layer and an external part of the usual width, that is inversely proportional to the square root of the Reynolds number /1/. The non-linearity of the equations of motion leads to the fact that on the external boundary of the Stokes layer a weak stationary motion is induced that for the exterior part of the BL manifests itself as the effect of slippage at the wall. The rate of slippage is proportional to the pressure gradient at the BL without pulsations i.e. directed downstream on a part of the adverse pressure gradient. Thus, we can expect that the presence of high-frequency pulsations in the flow increases the range of variation of angles of attack of a profile in which the flow around the leading edge proceeds without rupture of the BL. An equality is deduced that allows this range to be estimated.

**1. Statement of the problem. Equations for averaged flow.** Consider the flow round a thin profile with parabolic leading edge by a uniform flow of an incompressible fluid with velocity $U_\infty(1+\sigma\cos t)$, where $Tt/(2\pi)$ is the time, $T$ is the period and $\sigma$ is the amplitude of the velocity of the pulsations. We assume that the radius of curvature of the leading edge $L$ is small compared with the chord profile. We shall restrict ourselves to the analysis of the flow in the neighbourhood of the leading edge. For large Reynolds numbers $\mathrm{Re}=U_\infty L\nu^{-1}$ a narrow boundary layer is formed very close to the rigid surface. In a curvilinear orthogonal system of coordinates $Lx$, $\mathrm{Re}^{-1/2}Ly$, connected to the flow surface (Fig.1), the movement of the fluid at the BL is described by a traditional boundary value problem of the form

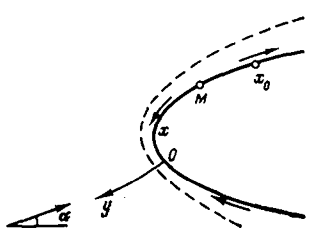


Fig.1

$$S^2\frac{\partial^2\psi}{\partial y\,\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\,\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=\frac{\partial^3\psi}{\partial y^3}+S^2\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}, \tag{1.1}$$
$$u=(1+\sigma\cos t)f(x;k)$$
$$\psi=\frac{\partial\psi}{\partial y}=0\quad(y=0);\quad \frac{\partial\psi}{\partial y}\to u\quad(y\to+\infty)$$

Here $\mathrm{Re}^{-1/2}LU_\infty\psi$ is the stream function and $S^2=2\pi LU_\infty^{-1}T^{-1}$ is the Strouhal number. The function $f(x;k)$ defines the velocity distribution at the exterior edge of the boundary layer in a flow without pulsations. The velocity on the exterior boundary depends both on the longitudinal coordinate $x$ and the dimensionless parameter $k$ that characterizes the degree of asymmetry of the flow and is linearly related to the angle of attack of the profile $\alpha$ (for an asymmetric profile with chord $b$ we have $k=\alpha(2b/L)^{1/2}$). In accordance with /2/, $f(x,k)=(z+k)(z^2+1)^{-1/2}$, where $z$ is the distance from the axis of the parabola to a point on the surface with coordinate $x$.

If $\sigma=0$, a regular solution of (1.1) exists if $0<k<k_0=1.1556$ /3/. If $k=k_0$ the solution has an extended singularity in the section $x=x_0=8.37$; if $k>k_0$, then the solution contains a Gol'dstein singularity /4/. Suppose the amplitude of the fluctuations is not zero and, furthermore, the Strouhal number is large. We shall assume that there exists a solution that is periodic with respect to time. With a view to reducing the volume of calculations we shall consider a typical range of strong fluctuations that holds if $\sigma=S\sigma_1$, $\sigma_1=O(1)$, $S\gg1$. In this case the solution of (1.1) at the main part of the BL $(y\sim1)$ can

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be represented in the form

$$\psi = S\sigma_1 y f \cos t + \psi_1 + S^{-1}\psi_2 + S^{-2}\psi_3 + O(S^{-3}) \tag{1.2}$$

Putting (1.2) in Eq.(1.1) we find that the general solution for the second term can be represented in the form $\psi_1 = \psi_s(x, y) + \psi_{11}(x, t)$. The function $\psi_{11}$ characterizes the displacing effect of the Stokes layer and will be found from the conditions for combining with a solution in the domain $y \sim S^{-1}$. To determine the stationary component of the solution $\psi_s$ it is necessary to consider the next terms in the expansion (1.2). The general solution of the equation for the third term has the form

$$\psi_2 = \psi_{20}(x, y) + \psi_{21}(x, t) + \sin t\, \psi_{22}(x, y)$$
$$\psi_{22} = \sigma_1\left(2yff' - f\frac{\partial\psi_s}{\partial x} - 2f'\psi_s + yf'\frac{\partial\psi_s}{\partial y}\right)$$

Here and later the derivative of a function with respect to its argument will be denoted by a prime. The functions $\psi_{20}$ and $\psi_{21}$ are still arbitrary at this stage of the solution.

For the next term of the expansion (1.2) we have the equation

$$\frac{\partial^2\psi_3}{\partial y\,\partial t} = \sigma_1 \cos t G[\psi_{20}] + \frac{1}{2}\sigma_1 \sin t G[\psi_{22}] + R[\psi_s] + \frac{\partial^2\psi_s}{\partial y^2}\frac{\partial\psi_{11}}{\partial x}$$
$$G[\psi_{2n}] = -f\frac{\partial^2\psi_{2n}}{\partial x\,\partial y} - f'\frac{\partial\psi_{2n}}{\partial y} + yf'\frac{\partial^2\psi_{2n}}{\partial y^2} \quad (n = 0, 2)$$
$$R[\psi_s] = \frac{\partial^3\psi_s}{\partial y^3} - \frac{\partial\psi_s}{\partial y}\frac{\partial^2\psi_s}{\partial x\,\partial y} + \frac{\partial\psi_s}{\partial x}\frac{\partial^2\psi_s}{\partial y^2} + ff'$$

Below it will be shown that the average value of the function $\psi_{11}$ over the period is zero. Therefore, the necessary and sufficient condition for the function $\partial\psi_3/\partial y$ to be periodic with respect to time takes the form $R[\psi_s] = 0$, i.e. the averaged motion in the main approximation is determined by the BL equations with the same pressure gradient as in a flow without fluctuations.

Expansion (1.2) will be uniformly suitable with respect to its longitudinal coordinate only in the case when the coefficients of the expansion and their derivatives of the necessary order are continuous in $x$. The least "dangerous" in this relation is the averaged motion, since it is described by a Prandtl number with given pressure gradient. The equation for the function $\psi_s$ requires specific boundary conditions. The condition as $y \to +\infty$ follows from (1.1) and has the form $\partial\psi_s/\partial y \to f(x; k)$. The conditions on the rigid surface are obtained as a result of combining (1.2) with the solution on the Stokes layer, on which $Y = Sy = O(1)$. **Here the stream** function can be represented in the form

$$\psi = \Psi_0(x, Y, t) + S^{-1}\Psi_1(x, Y, t) + O(S^{-2}) \tag{1.3}$$

The function $\Psi_0$ is a solution of the classical Stokes problem with a parametric dependence on the longitudinal coordinate

$$\Psi_0 = {}^1/_2\sigma_1 f\,[e^{it}\varphi_0 + \text{c. c.}], \qquad \varphi_0 = Y - i^{-1/2}(1 - \exp(-Yi^{1/2}))$$

The motion c.c. is used for a complex-conjugate expression. The combination of the solution on the Stokes layer with the solution in the domain $y \sim 1$ gives $\psi_{11} = -\sigma_1 f \cos(t - \pi/4)$. The problem for the next coefficient of expansion (1.3) has the form

$$\frac{\partial^3\Psi_1}{\partial Y^3} - \frac{\partial^2\Psi_1}{\partial Y\,\partial t} = \frac{\sigma_1^2}{4} ff'\{e^{2it}[\varphi_0'^2 - \varphi_0\varphi_0'' - 1] + [\varphi_0'\varphi_0^{*\prime} - \varphi_0^*\varphi_0'' - 1] + \text{c. c.}\} \tag{1.4}$$
$$\Psi_1 = \frac{\partial\Psi_1}{\partial Y} = 0 \quad (Y = 0); \qquad \Psi_1 = O(Y) \quad (Y \to +\infty)$$

Here $\varphi_0^*$ is the complex-conjugate function to $\varphi_0$. The solution of (1.4) is:

$$\Psi_1 = {}^1/_4\sigma_1^2 ff'\,[e^{2it}\varphi_{10}(Y) + \varphi_{11}(Y) + \text{c. c.}]$$

Since averaged motion is of most interest, we can restrict ourselves to considering the function $\varphi_{11}$. After transformations we obtain

$$\varphi_{11}' = -{}^3/_2 - 7i/2 + {}^1/_2(1 - i)\exp(-2^{1/2}Y) + (1 + 3i - Yi^{-1/2})\exp(-i^{1/2}Y) + i\exp(-(-i)^{1/2}Y)$$

Note that $\varphi_{11}' \to -(3 + 7i)/2$ as $Y \to +\infty$, i.e. a certain stationary flow is induced on the exterior boundary of the Stokes layer. Matching of the two-term expansion (1.3) with expansion (1.2) shows that the averaged motion on the main part of the BL must satisfy the slippage condition on the wall

$$\psi_s = 0, \quad \partial\psi_s/\partial y = -{}^3/_4\sigma_1^2 ff' \quad (y = 0)$$

Combining everything we have said above relating to the averaged flow in the domain $y \sim 1$, we arrive at the boundary value problem

$$\frac{\partial\psi_s}{\partial y}\frac{\partial^2\psi_s}{\partial x\,\partial y} - \frac{\partial\psi_s}{\partial x}\frac{\partial^2\psi_s}{\partial y^2} - ff' = \frac{\partial^3\psi_s}{\partial y^3} \tag{1.5}$$
$$\psi_s = 0, \quad \partial\psi_s/\partial y = -{}^3/_4\sigma_1^2 ff' \quad (y = 0)$$
$$\partial\psi_s/\partial y \to f(x; k) \quad (y \to +\infty)$$

The direction of the velocity of slippage is shown by arrows in Fig.1. The velocity of slippage becomes zero at the leading critical point $x = 0\,(f = 0)$, at the maximum point of the function $f$ (the point $M$) and asymptotically approximates to zero as $x \to \pm\infty\ (f \to \pm 1)$. Downstream with respect to the flow from the point $M$ we have $f' < 0$ (an unfavourable pressure gradient), therefore the slippage velocity in this part is directed towards the same side as the external flow.

Numerical solution of (1.5) presents serious difficulties because in the defined part of the BL the slippage velocity is directed along the opposite direction to the external flow. However, the problem becomes simpler if the effective fluctuation amplitude $\sigma_1$, and thus the slippage velocity also, are small.

**2. A boundary layer with slow slippage.** Suppose $\sigma_1 \ll 1$. We can expect that the flow at the BL remains unbroken if the angle of attack exceeds the critical value by an amount of the same order of smallness as that of the slippage velocity. Putting $k - k_0 = O(\sigma_1^2)$ we can represent the solution of (1.5) in the form

$$\psi_s = \Phi_0(x, y) + (k - k_0)\,\Phi_1(x, y) + \sigma_1^2\Phi_2(x, y) + O(\sigma_1^4)$$
$$f(x; k) = f_0(x) + (k - k_0)\,f_1(x) + O(\sigma_1^4)$$

The function $\Phi_0$ defines an extended singular solution at the BL without slippage /4/. For the next terms of the expansion we have the boundary value problems

$$\frac{\partial\Phi_0}{\partial y}\frac{\partial^2\Phi_i}{\partial x\,\partial y} + \frac{\partial^2\Phi_0}{\partial x\,\partial y}\frac{\partial\Phi_i}{\partial y} - \frac{\partial\Phi_0}{\partial x}\frac{\partial^2\Phi_i}{\partial y^2} -$$
$$\frac{\partial^2\Phi_0}{\partial y^2}\frac{\partial\Phi_i}{\partial x} - (f_0f_1)'\,\delta_{i1} = \frac{\partial^3\Phi_i}{\partial y^3}$$
$$\Phi_i = 0, \quad \partial\Phi_i/\partial y = -{}^3/_4 f_0 f_0'\delta_{i2} \quad (y = 0)$$
$$\partial\Phi_i/\partial y \to f_1\delta_{i1} \quad (y \to +\infty), \quad i = 1, 2$$
$$\delta_{ij} = 1 \quad (i = j); \quad \delta_{ij} = 0 \quad (i \neq j)$$

The solution of the boundary value problems for the functions $\Phi_0$ and $\Phi_1$ were studied in /4/. The problem for the function $\Phi_2$ has the solution:

$$\Phi_2 = -\frac{3}{4} f_0 f_0' \frac{\partial\Phi_0(x, y)}{\partial y} \Big/ \frac{\partial^2\Phi_0(x, 0)}{\partial y^2} \tag{2.1}$$

The last result can easily be generalized to the case of an arbitrary distribution of the slippage velocity with respect to a rigid surface.

All the functions $\Phi_i\ (i = 0, 1, 2)$ have a singularity in the section $x = x_0$, where the surface friction force in the main approximation is equal to zero. The nature of the singularities of the functions $\Phi_0$ and $\Phi_1$ is known /4/ and the corresponding result for $\Phi_2$ follows from (2.1). It turns out that if $\lambda_0 = f_0(x_0)f_0'(x_0)$, then as $x \to x_0 - 0$ at the BL any sublayer in which $\eta = y(x_0 - x)^{-1/4} = O(1)$ is separated. The representation of the solution in the sublayer has the form

$$\Phi_0 = {}^1/_6(x_0 - x)^{3/4}\lambda_0\eta^3 + {}^1/_2(x_0 - x)^{1/2}a_0\eta^2 + O((x_0 - x)^{5/4})$$
$$\Phi_1 = {}^1/_2(x_0 - x)^{-1/2}a_1\eta^2 + O((x_0 - x)^{1/4}$$
$$\Phi_2 = {}^3/_8(x_0 - x)^{-1/2}\lambda_0^2 a_0^{-1}\eta^2 + O((x_0 - x)^{1/4})$$
$$a_0 = 0{,}0085, \quad a_1 = -1{,}24$$

Note that the coefficient of the function $\Phi_2$ before the singular term depends on the slippage velocity only at the point $x_0$. The singularity in the solution is smoothed out in the small domain $x - x_0 = \sigma_1 x_1$, $y = \sigma_1^{1/4} y_1$, $(x_1, y_1) = O(1)$ where

$$\psi_s = {}^1/_6\sigma_1^{3/4}\lambda_0 y_1^3 + \sigma_1^{5/4} y_1^2 A(x_1) + \ldots$$

$$A(x_1) = \left({}^1/_4 a_0^2 x_1^2 + {}^3/_8 \lambda_0^2 + {}^1/_2 a_0 a_1 \sigma_1^{-2} (k - k_0)\right)^{1/2}$$

A solution can be obtained by the same method as in /4/. From the last relation, by virtue of the equality $\sigma = S\sigma_1$ it soon follows that there exists a solution that is regular for all value of $x_1$ if

$$k < k_0 + \frac{3\lambda_0^2}{4a_0 |a_1|} \left(\frac{\sigma}{S}\right)^2 \tag{2.2}$$

This inequality defines the range of variation of the angles of attack of the profile that allow a fluctuating stream to have an unbroken flow around the leading edge.

**3. Concluding remarks.** The main result of the paper, quantitatively expressed by relation (2.2), was obtained by means of a repeated passage to the limit in problem (1.1): initially it was assumed that $\sigma = S\sigma_1, \sigma_1 \sim 1, S \to \infty$, then, in the equation for a stationary component flow, $\sigma_1 \to 0$. In such an approach the validity of (2.2) is established only for values of $\sigma/S$ that are not too small. In fact, the range of validity for (2.2) is quite wide.

In this case, the the solution presented above is based on the fact that at the exterior boundary of the Stokes layer, the thickness of which is equal to $O(S^{-1})$, a stationary slippage rate of magnitude $O((\sigma/S)^2)$ is induced. The effect of slippage gives rise to a non-linear smoothing mechanism for a singularity in the solution for the problem of stationary motion in the main part of the BL. The non-linear domain has a length of $x - x_0 = O(\sigma/S)$ and in this case the thickness of any sublayer for the stationary component flow is equal to $O((\sigma/S)^{1/4})$. The scheme derived only holds in the case when the equations for averaging over time and for an oscillating component flow are separated. For this it is necessary that the thickness of any sublayer for the stationary part of the flow should be much larger than the thickness of the Stokes layer: $\sigma \gg S^{-3}$. Thus, for large values of the Strouhal number $S^2$ in the range $S^{-3} \ll \sigma \ll S$ the permissible variations of the angle of attack of the profile are defined by relation (2.2).

Under experimental conditions high-frequency pulsations in an inflowing stream are usually created by an acoustic disturbance of the field of flow (for example /5/). The effect of elimination of the flow separation that is observed in such cases from the leading edge of a profile is related to the generation of turbulence in the preseparated BL. Therefore, comparison of the theory derived above with experimental data is not justified. However, it is possible to detect a certain similarity between the physical mechanisms that lead to elimination of the rupture in turbulent and laminar flows. In both cases the action of Reynolds stresses has an effect, i.e. values averaged over time of paired derivatives, consisting of the pulsating components of the velocity vector. Under the influence of the Reynolds stresses a loading of the profile of average velocity occurs, as a result of which the BL is able to withstand a stronger increase in pressure.

The analysis that has been carried out can be extended to other forms of periodic flows of BLs for large Strouhal numbers: the flow round a profile with variable angle of attack, oscillations in the direction of a chord etc. Minor changes are required also in the case when the dependence of the parameters of the problem with respect to time are not single-harmonic.

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REFERENCES

1. TELIONIS D.P., Unsteady viscous flows, Springer, New York, 1981.
2. VAN DYKE M.D., Second-order subsonic airfoil theory including edge effects, NACA Report 1274, 1956.
3. WERLE M.J. and DAVIS R.T., Incompressible laminar boundary layers on a parabola at an angle of attack: a study of the separation point, Trans. ASME Ser. E.J. Appl. Mech., 39, 1, 1972.
4. RUBAN A.I., Singular solution of boundary layer equations which can be extended continuously through the point of zero surface friction, Izv. Akad. Nauk SSSR Mekh. Zhidkosti i Gaza, 6, 1981.
5. GINEVSKII A.S., VLASOV E.V. and KOLESNIKOV A.V., Aero-acoustic interaction, Mashinostroyenie, Moscow, 1978.

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